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Opinion

Solution of simultaneous equations of the form $p + q = \alpha$ and $pq = \beta$ using square formula method

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ABSTRACT: This paper presents a method for solving simultaneous equations of the form $\mathrm{p} + \mathrm{q} = \alpha$ and $\mathrm{pq} = \beta$ using the square formulae. The square formulae is a mathematical algorithm that can be applied to any system of linear and quadratic simultaneous equations, in this case of the form $\mathrm{p} + \mathrm{q} = \alpha$ and $\mathrm{pq} = \beta$. The paper begins by giving the overview of simultaneous equations and the square formula and its uses, followed by the derivation of the formulae. The paper then gave examples of the implementation of the square formulae and shows how it can be used to solve the given set of simultaneous \equations. Lastly, the paper presents further applications of the square formulae for solving other kinds system of simultaneous equations.

Keywords: Algorithm, new formulae, roots of equations, simultaneous equations methods, system of equations, variables.

INTRODUCTION

Simultaneous equations is an important topic in the algebra branch of Mathematics. There are times when you come across two or more unknown quantities and two or more equations relating to them. These are called simultaneous equations. When you are asked to solve such equations, you must find the values of the unknowns which satisfy all the given equations at the same time. They are simultaneous equations because the equations are solved at the same time. Simultaneous equations need algebraic skills to find the values of letters within two or more equations (Amemiya, 2013).

It is essential to recall that these equations include a set of a few independent equations, which is why simultaneous equations are also referred to as a system of equations, consisting of a finite set of equations. To solve the equations, we need to find the values of the variables that are part of these equations. The different types of mathematical equations are linear, quadratic and polynomial (Quandt, 1983). This is also applied to simultaneous equations in which one is linear, and the other is quadratic and is of the form.

$$\boldsymbol{a_1 x \pm b_1 y = c_1} \text{ and } \boldsymbol{a_2 xy = c_2}, \qquad (1)$$

Where $\boldsymbol{a_1, a_2, b_1}$ are coefficient constants and $\boldsymbol{c_1, c_2}$ are constants while $\boldsymbol{x}$ and $\boldsymbol{y}$ are the unknowns (variables).

A linear equation, statement that a first-degree polynomial, that is, the sum of a set of terms, each of which is the product of a constant and the first power of a variable, is equal to a constant.

Dass (2019) stated that an equation is a statement that uses the symbol of equality '=' to show that two quantities are equal. He furthered that a polynomial whose highest power of the unknown is 2 is called a quadratic equation. For example, $x^2 - 2x + 3 = 0$ is a quadratic equation.

Specifically, a linear equation in n variables is of the form $a_0 + a_1 x_1 + \ldots + a_n x_n = c$, in which $x_1, \ldots, x_n$ are variables, the coefficients $a_0, \ldots, a_n$ are constants, and c is a constant. If there is more than one variable, the equation may be linear in some variables and not in others. Thus, the equation $x + y = 3$ is linear in both *x* and *y*, whereas $x + y^2 = 0$ is linear in x but not in y. Any equation of two variables, linear in each, represents a straight line in Cartesian coordinates; if the constant term $c = 0$, the line passes through the origin.

A set of equations that has a common solution is called a system of simultaneous equations. For example, in the system $x + y = 5$ and $2x - y = 1$ both equations are satisfied by the solution $x = 2$, $y = 3$. The point $(2, 3)$ is the intersection of the straight lines represented by the two equations.

What is the fastest way to solve simultaneous equations?

To solve a set of simultaneous equations means to find the values of the unknowns. The values of the unknowns are called the roots of the equations. The fastest way to solve simultaneous equations really depends on the simultaneous equations that were given. If the coefficients in the simultaneous equations are set up in such a way that one of the variables can be easily eliminated, then the method of elimination could be considered to be the fastest. The graphing method will be the fastest one if the simultaneous equations are in slope-intercept form. But if the simultaneous equations are solved for one variable in terms of the other variable, then the substitution method would be considered the fastest (April and Joseph, 2022).

How do you solve simultaneous equations?

There are three common methods which have been used for a long time to solve simultaneous equations. The methods are as follows: elimination method, graphing method and substitution method. Though there are other methods that can be used through matrix, especially if the simultaneous equations involve more than two variables. Such methods are crammer’s rule method, the inversion method, the Gaussian elimination method, and so on. It is important to note that if any of the three methods were used to solve a system of simultaneous equations, the same result will be obtained. But based on the simultaneous equations, a method is considered to be easier and/or faster in solving the equations than the others (April, and Joseph, 2022).

In this paper, the main aim is to investigate a way of generating a formula to find a solution to simultaneous equations of the form $p + q = \alpha$ and $pq = \beta$ just as there is a formula for a quadratic equation of the form $ax^2 + bx + c = 0$.

Kalejaye (2011) defined a square as a rectangular shape with sides of equal length, and each angle is said to be a right angle. This work throws more light on the usage and benefit of the square in finding a solution to the above class of simultaneous equations.

April and Joseph (2022) stated that area measure is the surface measure of a particular shape, and that obtaining the area of a shape, like a square, means finding how much surface it covers. Park (1974) opined that the Babylonians solved equations by the use of geometrical diagrams, which always makes the process of solution practical and easier to understand. It is, however, noted that simultaneous linear equations only have a pair of solutions satisfying the two equations at the same time, while in the case where the simultaneous equations consist of one linear and one quadratic (cf (1)), the equations will have two pairs of solutions satisfying the two equations at the same time. For the case where two linear equations form a simultaneous equation, the process of getting a solution may not be so difficult, but where we have one linear and one quadratic (as in (1)), the solution always involves going through solving the associated quadratic equation.

ALGORITHM FOR THE DERIVATION OF A FORMULA FOR THE SQUARE METHOD

Consider the simultaneous equations

$$p + q = \alpha \quad \text{(i)}$$

And

$$pq = \beta \quad \text{(ii)}$$

Then an algorithm (a step-by-step process) for the square method of solution to the above classes of simultaneous equations can be developed. Draw a square of side α and divide each side into two equal parts, and name each part as p and q (Figure 1).

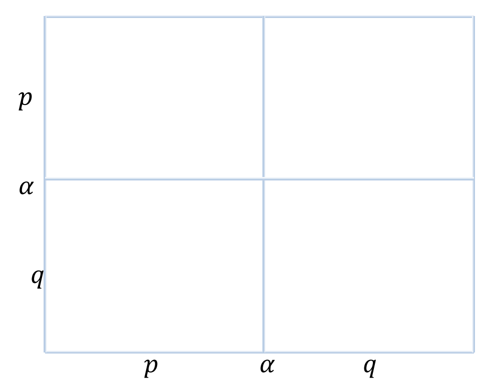

Figure 1. Square of side α and divide each side into two equal parts.

In Figure 1, $p = \frac{\alpha}{2}$ and $q = \frac{\alpha}{2}$. Hence,

$$p + q = \frac{\alpha}{2} + \frac{\alpha}{2} = \alpha$$

which satisfied equation (i), but not suitable for (ii). So, there is a need to choose a small displacement from α (say n) such that $p = \frac{\alpha}{2} + n$ and $q = \frac{\alpha}{2} - n$ (Figure 2).

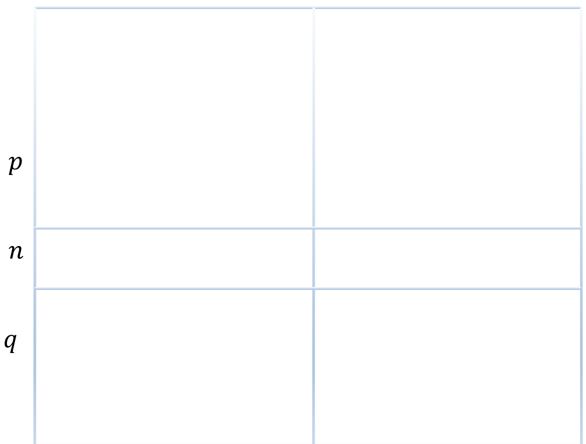

Figure 2. Square of side α and divide each side into two equal parts with a small n displacement.

In fact, the information in Figure 2 satisfied both of the equations. Now consider equation (ii), we see that:

$$\left(\frac{\alpha}{2}+n\right)\left(\frac{\alpha}{2}-n\right)=\beta$$

$$\left(\frac{\alpha}{2}\right)^2-n^2=\beta$$

$$n^2=\left(\frac{\alpha}{2}\right)^2-\beta$$

$$n=\pm\sqrt{\left(\frac{\alpha}{2}\right)^2-\beta}$$

Thus,

$$p=\frac{\alpha}{2}+n=\frac{\alpha}{2}\pm\sqrt{\left(\frac{\alpha}{2}\right)^2-\beta} \qquad (2)$$

And

$$q=\frac{\alpha}{2}-n=\frac{\alpha}{2}\mp\sqrt{\left(\frac{\alpha}{2}\right)^2-\beta} \qquad (3)$$

Remember now that in a system of simultaneous equations, p could be related directly to the variable x while q could be treated as a variable y. So, using any of the equations (2) and (3) to obtain the values of an unknown will help in determining the values of the other variable.

Existing solution

The conventional methods for solving simultaneous equations have been discussed above, which are the elimination method, the substitution method and the graphing method. In this section, we shall discuss the relationship between the substitution method and the square method.

Illustration 1: Solve the simultaneous equations:

$x+y=10$ (i)
$xy=21$ (ii)

Solution
By the substitution method

From eqn (i) $x=10-y$ (iii)

Using eqn (iii) in eqn (ii), we have

$$(10-y)y=21$$

$$10y-y^2=21$$

$$y^2-10y+21=0$$

Factorising the above gives

$$(y-3)(y-7)=0$$

which gives that $y=3$ or $y=7$.

Now, substituting the values of y in (iii), we have the values of x to be $x=7$ or $x=3$. With these, we have the roots of the equations in coordinate form as follows

$(x_1,y_1)=(7,3)$ or $(x_2,y_2)=(3,7)$

Square formulae

Here we shall compare the roots of the equations gotten from the existing solution with those of the square formulae; if they are equal, then we confirm the method is valid. Now, given the equations:

$x+y=10$ (i)
$xy=21$ (ii)

Compare the equations above with

$p+q=\alpha$ (i)
$pq=\beta$ (ii)

Here $=p$, $y=q$, $\alpha=10$, $\beta=21$. Now using (2), we have that

$$p=\frac{10}{2}\pm\sqrt{\left(\frac{10}{2}\right)^2-21}$$

$$=5\pm\sqrt{25-21}$$

$= 5 \pm 2$

$\therefore\ p = 7, p = 3$

In the same manner, using (3), it can be easily seen that

$q = 3$, or $q = 7$

which gives that $(x_1, y_1) = (7{,}3)$ or $(x_2, y_2) = (3{,}7)$ as before.

Illustration 2: Solve the simultaneous equations, using the square formulae

$x + y = 6$ (i)
$xy = 8$ (ii)

Using (2), we have

$$p = \frac{6}{2} \pm \sqrt{\left(\frac{6}{2}\right)^2 - 8}$$

which gives that $p = 4$, or $p = 2$.

Also, using (3) gives the values of q as 2 or 4

Conclusion

In conclusion, the results proved in this paper have shown new formulae for solving simultaneous equations of the form $p + q = \alpha$ and $pq = \beta$ whose the coefficients of the variables are unit. It was discovered, in the course of study, that the formula is suitable for equations with positive values since the formulae related to the length of a square and the length of a square can never be negative, while the research is still ongoing on how to propose a method to solve the kinds of equations whose coefficients are negative.

The method discussed in this paper with the formulae $p = \frac{\alpha}{2} + n = \frac{\alpha}{2} \pm \sqrt{\left(\frac{\alpha}{2}\right)^2 - \beta}$ and $q = \frac{\alpha}{2} - n = \frac{\alpha}{2} \mp \sqrt{\left(\frac{\alpha}{2}\right)^2 - \beta}$ can be used to solve any kind of quadratic simultaneous equations, even if the coefficients of the unknowns are not unit, but that will be discussed in our next paper. The paper has given the parents, the teachers, the students and the education planners an eye opener that new methods can be proposed to solve simultaneous quadratic equations and any other Mathematics topics.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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