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Full Length Research

Comparative study of heat and mass transfer efficiency of water base and ethylene glycol base nanofluid in MHD and spherically enclosed system

Egbo Chijioke*, Ngiangia Alalibo and Obong Hilary

Department of Physics, University of Port Harcourt, Port Harcourt, Choba, Rivers State, Nigeria.

*Corresponding author. Email: chijioke_egbo@uniport.edu.ng; +234806794755; ORCID: 0000-0003-3598-9982.

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ABSTRACT: The analytic study of ethylene glycol base and water base tin oxide nanofluids in an MHD spherical enclosure using combined models of thermal conductivity and effective viscosity was carried out based on the optically thick medium of the Rosseland radiative heat flux approximation. The governing equation was non-dimensionalized and solved using the Laplace Transform Technique for Sherwood number, Nusselt number, and Skin friction from the respective solution of concentration, energy, and velocity equations. Results of immense engineering importance regarding how energy and mass transfer efficiency in a spherically enclosed system depends on the Prandtl number, parameter of radiation, how drag forces in spherically enclosed system can be related to Schmidt number, parameter of chemical reaction, and how the viscous resistance within fluid boundary layer of a spherically enclosed system is characterized base on the dimensionless hydrodynamic parameters as well as the nanoparticles volume fraction of the novel nanofluid samples considered.

Keywords: Ethylene glycol, tin oxide, nanofluid, water, Nusselt number, Sherwood number, skin friction.

INTRODUCTION

Numerous studies have highlighted the anomalous energy transfer efficiency offered by nanofluid in different geometrical systems found in various areas of engineering applications. In renewable energy engineering. Loni *et al.* (2018a, 2018b) studied the thermal performance of nanofluid in dish concentrator where they used carbon nanotube/thermal oil, where results of the thermal efficiency of Al2O3-oil and SiO3-oil were compared in experimental study. Solomon *et al.* (2017) investigated how different aspect ratio of geometrical square cavities affects heat transfer through natural convection using Al2O3-water based nanofluid. Saghir *et al*. (2016) compared the single-phase model with 2 two-phase models in the same square cavity using a numerical model. The energy transfer efficiency of nanofluid has also been observed to be influenced by a certain percentage of nanoparticle volume fraction, which will correspondingly influence the viscosity and thermal conductivity as well as the density of the base fluid.

When a flow is considered in a complex system, Chamkha *et al.* (2012) presented a graphical result of Nusselt and Sherwood number of nanofluid flowing over a porous vertically embedded cone under steady state, mixed convective boundary layer and laminar flow condition, having considered the Rosseland approximation for radiative energy flux, Brownian motion and thermophoretic activities were incorporated at constant concentration of nanoparticle volume fraction and temperature. While Salari *et al*. (2014) highlighted observations of the relationship between Brownian motion, Lewis number, Thermophoresis number, Prandtle number with Nusselt number and Sherwood number in their study of heat transfer that accompany nanofluid as it flows through a stretching plate when nanofluid's volume fraction and temperature is varied, and both kept constant are the two cases of the boundary condition of the study. Makinde and Aziz (2011) also considered nanofluid flow through a stretching sheet in a numerical evaluation of induced boundary layer in a convective heating condition, the Lewis number, Prandtl number, thermophoresis number, parameter of Brownian motion and convective biot number are the parameters that determines the solution of nanoparticle distribution and temperature boundary layer. In a similar case, Khan and Pop (2010) observed that as the independent dimensionless hydrodynamic parameters increases, the Nusselt number is enhanced, while the Sherwood is enhanced for higher values of the Prandtl number (Ahmad *et al*., 2016) in the study where nanofluid flow through vertical Riga plate in a boundary layer of mixed convective flow under the influence of strong suction force. In their study, they also considered thermophoretic and Brownian motion conditions in developing the mathematical model that represents the flow condition. The study of the dependency of natural convection on heat and mass transfer in nanofluid was numerically carried out to ascertain the influence of water-based copper nanofluid. Profiles of concentration, velocity, and temperature were presented with a graphical aid, and skin friction, Nusselt number, and Sherwood number were also presented (Sheri and Thumma, 2016). With the introduction of convective mass transfer, the two-dimensional magnetohydrodynamic (MHD) boundary layer flow in nanofluid was investigated in the presence of Brownian motion, induced magnetic field, and thermophoresis. The Nusselt number, Sherwood number, and skin friction observed from the wall alongside were part of the results obtained from their nonlinear differential equation developed for the system under consideration. In flow triggered by natural convection of nanofluid over vertical plate with constant heat flux (Khan and Aziz, 2011), in another study the natural convective flow due to buoyancy force within boundary layer were considered (Aziz and Khan, 2012), and based on regression data the Nusselt number and Sherwood number were obtained via correlation. A three-dimensional MHD flow situation considered in the presence of radiative thermal energy flux carried out by Shehzad *et al*. (2014), and data of numerical computation were obtained for Sherwood number and Nusselt number from the mathematical expression who's independent parameters includes the ratio of relaxation to retardation time, induced magnetic field, stretching rate ratio, Lewis number, Prandtle number, radiation term, Brownian motion and thermophoretic (Upadhyay *et al*., 2021). Nusselt number and other engineering parameters have been computed in the study of nanofluid heat transfer for medium temperature for solar panel application. The heat transfer effects of a copper water and alumina oxide hybrid nanofluid (Suresh *et al.,* 2012) completely developed laminar convective heat transfer and pressure drop properties employing a hybrid nanofluid of copper water and alumina oxide through a circular tube that is heated evenly. copper nanocomposite powder and alumina oxide using a thermochemical process that includes a hydrogen reduction approach. The resulting hybrid nanopowder was then dissolved in deionized water to create a stable hybrid nanofluid with a volume concentration of 0.1%. Natural convection in enclosures is impacted by the varied characteristics of nanofluids (Abu-Nada *et al.,* 2010). Variable thermal conductivity and variable viscosity of alumina water and copper oxide water nanofluids are used to promote heat transfer in a differentially heated enclosure. The results show that the volume fractions of nanoparticles, aspect ratios, and Rayleigh numbers were all kept at a range of values. At low Rayleigh numbers, it was found that the Nusselt number was insensitive to the volume fraction. The average Nusselt number was found to be more responsive to the viscosity models than to the thermal conductivity models at high Rayleigh numbers. It is also shown that the aspect ratio affects the Nusselt number. Under turbulent flow conditions, the heat transfer performance of titanium oxide and alumina oxide water nanofluids across an annular channel with a constant wall temperature boundary condition is examined in Nasiri *et al*. (2011). According to the experimental findings, the Nusselt number of nanofluids is greater than that of the base fluid for a given Peclet number. For both used nanofluids, the enhancement rises as the concentration of nanoparticles increases. The investigation's findings indicate that there is no discernible variation in the augmentation of heat transmission between the two used nanofluids. Various studies on heat transfer enhancement of different strains of nanofluid have been carried out using experimental approach of Arani and Amani (2012) and Mangrulkar and Kriplani (2013), where titanium oxide nanoparticle volume fraction, Reynolds number, and other parameters were varied to observe the impact of their influence on the Nusselt number. Numerical analysis has also been used (Sheikholeslami and Ganji, 2013; Karimipour, 2015; Zerradi *et al*., 2014; Motahari and Barati, 2019; Nasrin *et al.,* 2012) to study copper oxide nanoparticle and other non-dimensionalized hydrodynamic parameters in water base fluid considering various boundary condition and coordinate system, as well as selected models of effective thermal conductivities and viscosities.

Therefore, this study aims to numerically analyze and compare the transfer efficiency of Tin oxide water base (SnO_2-H_2O) and Tin oxide ethylene glycol base (SnO_2-$C_2H_6O_2$) nanofluids in magnetohydrodynamic (MHD) spherical enclosure using combined models of proposed effective thermal conductivity and viscosity models and the Rosseland approximation for optically thick medium.

MATERIALS AND METHODS

Then, as the water base and the ethylene glycol base tin oxide nanofluid flow with a velocity whose radial component is V_r in an MHD and spherically enclosed system whose length *L,* and radius *r* are represented*.* If the Continuity equation, Momentum equation, Energy equation, and Concentration equation are the sets of partial differential equations in spherical coordinate system is the governing equation of the system. Also, when the fluid is incompressible, the flow situation is also considered to be in a steady state throughout the flow process in the spherically enclosed system. Hence, the governing partial differential equations representing the description of the fluid flow conditions in the system are respectively given by

$$\frac{\partial}{\partial}(r^2 V_r) = 0 \qquad 1$$

$$\left(V_r \frac{\partial V_r}{\partial r}\right) = \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 V_r}{\partial r^2} + \frac{4}{r}\frac{\partial V_r}{\partial r} + \frac{2V_r}{r^2}\right) + g\beta'(C - C') + g\beta(T - T') - \frac{\sigma B_0^2 V_r}{\rho_{nf}^2} \qquad 2$$

$$\left(V_r \frac{\partial T}{\partial r}\right) = \frac{k_{nf}}{(\rho C_P)_{nf}}\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right) - \frac{1}{(\rho C_p)_{nf}}\left(r\frac{\partial q_r}{\partial r} + \frac{q_r}{r^2}\right) \qquad 3$$

$$\left(V_r \frac{\partial C}{\partial r}\right) = \frac{D}{(\rho C_p)_{nf}}\left(\frac{\partial^2 C}{\partial r^2} + \frac{2}{r}\frac{\partial C}{\partial r}\right) - \frac{1}{(\rho C_p)_{nf}}\frac{1}{r^2}(K_\infty^2 C) \qquad 4$$

Where ρ_{nf} is the density of nanofluid, *C* is the concentration of the fluid, *r* is the radial distance of the spherical coordinate, V_r is the radial component of the flow velocity, μ_{nf} is the resultant viscosity of nanofluid, *g* is the gravitational term, *σ* is the parameter of electrical conductivity, and B_o is the magnetic term, k_{nf} is the resultant thermal conductivity term, *T* is the nanofluids temperature, q_r is the radiation heat flux vector, *D* is the chemical diffusivity, and k_∞ is the parameter of chemical reaction, $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid, *β* is the thermal expansion due to temperature, *β'* is the thermal expansion due to concentration of nanofluid,. Subjected to the boundary condition given by

$$V_r = V_W, T = T_W, and\ C = C_W \qquad 5$$

As

$$r = 0, T \to 0, C \to 0\ and\ t = 0$$

The term q_r from equation 3 represents the radial effect of the radiative heat flux, which, according to the Rosseland approximation for optically thick model, the equation of diffusivity of radiative energy flux when the thermal layer becomes so thick is given by

$$q_r = \frac{4K_B}{3\alpha}\frac{\partial T^4}{\partial r} \qquad 6$$

Where K_B is the Stefan-Boltzmann constant and *α* is the absorption coefficient, in an instance where we assume that the temperature difference within the flow is sufficiently small such that T^4 can be approximated using the Taylor series to carry out the required expansion about T_∞.

$$T^4 = T_\infty^4 - 4T_\infty^3(T - T_\infty) + 6T_\infty^2(T - T_\infty)^2 \qquad 7$$

neglecting the terms of higher order, and simplifying the expansion, we obtain.

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \qquad 8$$

Considering equation (6) and (8),

$$\frac{\partial q_r}{\partial r} = \frac{16K_B T_\infty^3}{3\alpha}\frac{\partial^2 T}{\partial r^2} \qquad 9$$

Our governing equation (3) is transformed into

$$\left(V_r \frac{\partial T}{\partial r}\right) = \frac{k_{eff}}{(\rho C_P)_{nf}}\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right) - \left(-\frac{16K_B T_\infty^3 r}{3\alpha(\rho C_p)_{nf}}\frac{\partial^2 T}{\partial r^2} + \frac{r}{(\rho C_p)_{nf}} q_r\right) \qquad 10$$

For the effective viscosity term in our momentum equation, combined models proposed by Einstein (1956) and Batchelor (1977) was used, having considered both the non-spherical and spherical nature of the Tin oxide nanoparticle of the study, the combined of model of the viscosity is given by

$$\frac{\mu_{eff}}{\mu_{bf}} = \left(1 + 2.5\emptyset_{np}\right) + \left(1 + 2.5\emptyset_{np} + 6.2\emptyset_{np}\right) \qquad 11$$

Where $\emptyset_{np}$ is the nanoparticle volume fraction, μ_{eff} is the effective dynamic viscosity of the nanofluid, μ_{bf} is the dynamic viscosity of the base fluid.

Also, the effective thermal conductivity terms valid for both spherical and non-spherical shape nanoparticles proposed by Zerradi *et al.* (2014) was used, which has the Brownian motion term and is combined with the effective model proposed by Bhattacharya et al. (2004) and Koo and Kleinstreuer (2004, 2005b).

$$\frac{k_{eff}}{k_{bf}} = \emptyset_{np} + \left(1 - \emptyset_{np}\right) + (5 \times 10^4)\beta\emptyset_{np}(\rho C_{np})_{np}\sqrt{\frac{K_B T}{\rho_p D}} f(T, \emptyset_{np}) \qquad 12$$

where (ρ_{nf}) is the density of nanofluid, (β'_{nf}) is the thermal expansion due to concentration of nanofluid, (β_{nf}) thermal expansion due to temperature of nanofluid, and (C_p) is the heat capacity proposed by Nasrin *et al.* (2012) in their study of heat transfer augmentation in a two-sided lid-driven differential heat cavity utilizing nanofluid, and Motahari and Barati (2019) in their study of the exact solution for free convective flow of nanofluid with ramped wall temperature. The following are the parameters obtained:

$$\frac{\rho_{nf}}{\rho_{bf}} = \left(1-\emptyset_{np}\right) + \emptyset_{np}\frac{\rho_{np}}{\rho_{bf}} \quad 13$$

$$\frac{\beta_{nf}}{\beta_{bf}} = \left(1-\emptyset_{np}\right) + \emptyset_{np}\frac{\beta_{np}}{\beta_{bf}} \quad 14$$

$$\frac{\beta'_{nf}}{\beta'_{bf}} = \left(1-\emptyset_{np}\right) + \emptyset_{np}\frac{\beta'_{np}}{\beta'_{bf}} \quad 15$$

$$\frac{(C_p)_{nf}}{(C_p)_{bf}} = \left(1-\emptyset_{np}\right) + \emptyset_{np}\frac{(C_p)_{np}}{(C_p)_{bf}} \quad 16$$

where the nanoparticle volume fraction $\emptyset_{np}$ is given by the expression

$$\emptyset_{np} = m\frac{\pi}{6}D_s \quad 17$$

where *m* is the number of particles per unit volume, D_s is the average diameter of the particle.

Dimensional analysis

Following the Buckinham-π-theorem, the dimensional homogeneity that will enable us tackle the problem of our governing equation is stated as

$$u = \frac{v_r t'}{r'}, r = \frac{r'}{d}, t = \frac{r'}{u't'}, k_o = \frac{K_r^2\left(\rho C_p\right)_{bf}}{V'^2\left(\rho C_p\right)_{nf}}, Pr = \frac{k_f\left(\rho C_p\right)_{nf}}{k_{nf}\left(\rho C_p\right)_{bf}}, Ha = \frac{\sigma B_o^2\mu_{nf}}{\rho_{nf}^2 V^2}$$

$$Gr_\theta = \frac{g\beta(T-T_o)\mu_{nf}}{Vu'^2_o}, Gr_C = \frac{g\beta'^{(C-C_o)\mu_{nf}}}{Vu'^2_o}, Re = \frac{\mu_f\rho_{nf}}{\mu_{nf}\rho_f}, N = \frac{16\zeta T_o^3\rho_f}{3\alpha\left(C_p\right)_{nf}}$$

$$Sc = \frac{k_f\left(\rho C_p\right)_{nf}}{D\left(\rho C_p\right)_{bf}}, \theta = \frac{T-T_o}{T_o} \quad 18$$

where *Sc* is the Schmidt number, *Pr* is the Prandtl number, Gr_c is the concentration Grashof's number, Gr_θ is the thermal Grashof's number, *Ha* is the magnetic Hartmann number, *N* is the dimensionless radiation term, k_o is the dimensionless chemical reaction term, *Re* is the Reynolds number and *θ* is the dimensionless temperature term, *r* is the dimensionless radial distance, *t* is the dimensionless time, and *u* is the dimensionless velocity.

The dimensionless governing equation becomes

$$\frac{\partial}{\partial r}\left(r^2 u\right) = 0 \quad 19$$

$$u\frac{\partial u}{\partial r} = A_1 Re^{-1}\left(\frac{\partial^2 u}{\partial r^2} + \frac{4}{r}\frac{\partial u}{\partial r} + \frac{2u}{r^2}\right) + A_2 Gr_c C + A_3 Gt_\theta\theta + A_4 Hau \quad 20$$

$$u\frac{\partial\theta}{\partial r} = A_5 Pr^{-1}\left(\frac{\partial^2\theta}{\partial r^2} + \frac{2}{r}\frac{\partial\theta}{\partial r}\right) + A_6 N\left(\frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{\theta}{r^2}\right) \quad 21$$

$$u\frac{\partial C}{\partial r} = A_6 Sc^{-1}\left(\frac{\partial^2 C}{\partial r^2} + \frac{2}{r}\frac{\partial C}{\partial r}\right) + A_6\frac{1}{r^2}k_o C \quad 22$$

where

$$A_1 = \frac{\mu_{nf}/\mu_f}{\rho_{nf}/\rho_f} = \frac{\left(1+2.5\emptyset_{np}\right)+\left(1+2.5\emptyset_{np}+6.2\emptyset_{np}\right)}{1-\emptyset+\emptyset^{\rho_{np}}/\rho_{bf}} \quad 23$$

$$A_2 = 1-\emptyset_{np} + \emptyset_{np}{}^{\beta'_{np}}\Big/_{\beta'_{bf}} \quad 24$$

$$A_3 = 1-\emptyset_{np} + \emptyset_{np}{}^{\beta_{np}}\Big/_{\beta_{bf}} \quad 25$$

$$A_4 = 1-\emptyset_{np} + \emptyset_{np}{}^{\rho_{np}}\Big/_{\rho_{bf}} \quad 26$$

$$A_6 = \frac{1}{1-\emptyset_{np}+\emptyset_{np}{}^{\left(\rho C_p\right)_{np}}\Big/_{\left(\rho C_p\right)_{bf}}} \quad 27$$

$$A_5 = \frac{\frac{k_{np}+2k_{bf}+2\emptyset_{np}\left(k_{bf}-k_{np}\right)}{k_{np}+2k_{bf}-\emptyset_{np}\left(k_{bf}-k_{np}\right)} + \left(5\times10^4\right)\beta\emptyset_{np}\left(\rho C_p\right)_{np}\sqrt{\frac{K_B T}{\rho_p D}}f(T,\emptyset)}{1-\emptyset_{np}+\emptyset_{np}{}^{\left(\rho C_p\right)_{np}}\Big/_{\left(\rho C_p\right)_{bf}}} \quad 28$$

Integrating equation 19.

$$u = \frac{c}{r^2} \quad 29$$

where *c* is a constant. and if *c = 1,* and the value substituted for *u* into our governing equations, which were further simplified by multiplying all through by r^2, and rearranging the equations for them to be equated to zero.

$$\frac{A_1}{Re}\left(r^2\frac{\partial^2 u}{\partial r^2} + 4r\frac{\partial u}{\partial r} + 2u\right) - \frac{\partial u}{\partial r} + A_2 r^2 Gr_c C + A_3 r^2 Gt_\theta\theta + A_4 r^2 Hau = 0 \quad 30$$

$$\frac{A_5}{Pr}\left(r^2\frac{\partial^2\theta}{\partial r^2} + 2r\frac{\partial\theta}{\partial r}\right) - \frac{\partial\theta}{\partial r} + A_6 N\left(r\frac{\partial\theta}{\partial r} + \theta\right) = 0 \quad 31$$

$$\frac{A_6}{Sc}\left(r^2\frac{\partial^2 C}{\partial r^2} + 2r\frac{\partial C}{\partial r}\right) - \frac{\partial C}{\partial r} + A_6 k_o C = 0 \quad 32$$

Solution techniques

The simplified nondimensionalized governing equation of the system under consideration in this study is solved using the Laplace Transform Technique, and are transformed back into the radial dependent sets of

Table 1. The thermophysical properties of the individual fluid and solid materials.

Materials of nanofluid	Density (kg/m^3)	Viscosity (Pa*s)	Thermal Conductivity ($Wm^{-1}K^{-1}$)	Specific Heat Capacity (J/kgK)
Tin Oxide (SnO_2)	6950	0.0035	8.95	686.2
Ethylene Glycol ($C_2H_6O_2$)	1082	0.048	0.253	3140
Water (H_2O)	997	0.001	0.613	4200

Table 2. Numerical values of parameter of our nondimensionalizing coefficients.

Material Parameter/Properties	Numerical value (K^{-1})
Thermal expansivity of water (β_W)	2.1×10^{-4}
Thermal expansivity of ethylene glycol (β_{EG})	3.3×10^{-3}
Thermal expansivity of Tin Oxide (β_{np})	2.4
Concentration expansivity of water (β'_W)	8.5
Concentration expansivity of ethylene glycol (β'_{EG})	2.6
Concentration expansivity of Tin Oxide (β'_{np})	0.5
Diffusivity of water (D_W)	2.23×10^{-6}
Diffusivity of ethylene glycol (D_{EG})	5.67×10^{-8}
Stefan-Boltzmann constant (k_B)	1.38×10^{-23}
B(selected based on the nature of the nanoparticle)	$0.0011(100\phi_p)^{-0.7272}$

Table 3. Selected values of our nondimensionalize parameters.

Re	Grc	Gt	Sc	Ko	γ	N	Pr	Ha	Φ
100	2.0	2.5	2.5	5.0	0.96	2.5	4.0	10.0	0.02

equation using the Inverse Laplace Transform for realistic values of parameters presented in Tables 1, 2, and 3.

$$\bar{C}(s) = \frac{\frac{2A_6}{Sc}\left(\frac{1}{s^3}+\frac{1}{s^2}\right)-1}{\left(\frac{A_6}{Sc}\left(\frac{4}{s}\right)-s+A_6k_o\right)} \quad 33$$

$$\bar{\theta}(s) = \frac{\left[\left(\frac{1}{s^3}+\frac{1}{s^5}+\frac{1}{s^2}+\frac{1}{s^4}\right)\frac{\gamma}{Pr}\right]-1-\frac{A_6N}{s^2}}{\left[\frac{2\gamma}{Pr}\left(\frac{1}{s}+\frac{1}{s^3}\right)-s+A_6N\left(\frac{1}{s}+1\right)\right]} \quad 34$$

$$\bar{U}(s) = \frac{\frac{2A_1}{Re}\left(\frac{1}{s^3}+\frac{2}{s^2}\right)-1-A_2Gr_c\frac{2}{s^3}\left[\frac{\frac{2A_6}{Sc}\left(\frac{1}{s^3}+\frac{1}{s^2}\right)-1}{\left(\frac{A_6}{Sc}\left(\frac{4}{s}\right)-s+A_6k_o\right)}\right]-A_3Gt_\theta\frac{2}{s^3}\left(\frac{\left[\left(\frac{1}{s^3}+\frac{1}{s^5}+\frac{1}{s^2}+\frac{1}{s^4}\right)\frac{\gamma}{Pr}\right]-1-\frac{A_6N}{s^2}}{\left[\frac{2\gamma}{Pr}\left(\frac{1}{s}+\frac{1}{s^3}\right)-s+A_6N\left(\frac{1}{s}+1\right)\right]}\right)}{\left(\frac{2A_1}{Re}\left(\frac{3}{s}+1\right)-s+\frac{2A_4Ha}{s^3}\right)} \quad 35$$

Engineering applications

Skin friction for engineering applications, including enhancing the efficiency of vehicles and describing surface flows, is crucial. Additionally, it is a technology that facilitates the development of new industries like flow control. Furthermore, since a decrease in drag during cruise results in a decrease in fuel consumption, the measurement of skin friction is important to the aircraft industry. Another crucial metric for describing the condition of a turbulent boundary layer is skin friction, which is crucial for both helping to manage these flows and for gaining a basic knowledge of them. In a propeller wall, decreasing skin friction causes loss from shear force caused by turbulent friction

$$\left.\frac{dU(r)}{dr}\right|_{r=0} \quad 36$$

Sherwood number indicates the mass transfer coefficient for transporting mass from a fluid to a swarm of spherical particles. It is a dimensionless quantity. It is utilized in the engineering aspects of catalytic converter designs for exhaust gas cleaning and high-pressure process technology. Additionally, it is provided by

$$-\left.\frac{dC(r)}{dr}\right|_{r=0} \quad 37$$

Nusselt number characterizes the convective heat transfer between a fluid, whether it is a solid surface, a liquid or gas, and improvements in heat intensification methods for shell and tube heat exchangers. Calculating the amount of heat exchanged between the nanofluid and its boundaries is crucial to the dynamics of viscous nanofluids. It is given by;

$$-\left.\frac{d\theta(r)}{dr}\right|_{r=0} \quad 38$$

RESULTS AND DISCUSSION

Numerical values of the Sherwood number for varying Schmidt numbers for the water base and ethylene glycol base nanofluid are presented in Table 4. The results are obtained at constant selected values of the chemical reaction term and nanoparticle volume fraction. The result presented shows a constant Sherwood value as the Schmidt number increases. The Sherwood number presented for the ethylene glycol base nanofluid is relatively higher than the Sherwood number presented for the water base – Tin oxide nanofluid.

The physical implication of the result presented is that the total mass transfer of our nanofluid constituent particles is mainly through convection, and this convective mass transfer is irresponsive to change in the Schmidt number in both the water-based and ethylene glycol-based nanofluid in spherical enclosure. Further implication of the result is that the total mass transfer rate through convection is higher in ethylene glycol base fluid than in water base fluid because ethylene glycol base fluid has a relatively higher density than water. Results presented by Kumar *et al.* (2022) show that the mass transfer rate defined by the Sherwood number responds greatly to changes in the Schmidt number; the reason could be the appearance of the magnetic field term in their model.

Presented in Table 5 are the numerical results of the Sherwood number for varying chemical reaction terms of the water base and ethylene glycol base nanofluid, respectively. While Schmidt number and nanoparticle volume fraction is kept constant, it was observed from the result that the Sherwood number increases as the parameter of chemical reaction increases. The result presented in Table 5 shows that the Sherwood number obtained for the ethylene glycol base nanofluid is greater than the value of the result obtained for the water base nanofluid as the value of the Sherwood number extends to the third decimal place.

The physical implication of the result is that mass transfer by convection dominates diffusive mass transfer, and the domination of the mass transfer by convection increases as the parameter of chemical reaction increases. The results further imply that the diffusive mass transfer vanished gradually as the parameter of chemical reaction increased. Also, mass transfer through convection is higher in ethylene glycol base fluid; it is observed in the water base nanofluid, and this could be due to the difference in their density. The result here is in excellent agreement with the result obtained by Rout and Mishra (2018).

The result of the Sherwood number for varying nanoparticle volume fraction is presented in Table 6 for the water-based and ethylene glycol-based nanofluid, respectively. The result presented shows that as the nanoparticle volume fraction increases, the Sherwood number decreases. The result of the Sherwood number obtained for the ethylene glycol base nanofluid is relatively higher compared to the Sherwood number that was obtained for water base nanofluid from the third decimal place.

The physical implication of the results presented in Table 6 for water base and ethylene glycol base nanofluid respectively, is that convective mass transfer dominates diffusive mass transfer and the domination of the convective mass transfer decrease as nanoparticles volume fraction increases. This further implies that the effect of diffusive mass transfer becomes more significant as the number of nanoparticles per unit volume increases in the base fluid. Also observed, as a physical implication of the result, is the relatively higher convective mass transfer rate presented in Table 6 for ethylene glycol base nanofluid when compared to the mass transfer rate by convection observed in Table 12 for the water base nanofluid. In the work of Patil *et al*. (2018), they observed that variation in the concentration of nanoparticles did not trigger any change in mass transfer rate at the wall with surface roughness, which makes the result in this study contrary to the result they obtained.

The result of the Nusselt number when Brownian motion is taken into account, when Brownian motion is not taken into account is presented in Table 7 for the water base and ethylene glycol base nanofluid, when Prandtl number is varied. The result presented shows

Table 4. Sherwood number of nanofluid for varying Schmidt number when $k_o = 5$ and $\varphi = 0.02$

Parameters	Water	Ethylene glycol
Sc	*Sh*	*Sh*
2.5	4.90749	4.90923
3.5	4.90749	4.90923
4.5	4.90749	4.90923
5.5	4.90749	4.90923

Table 5. Sherwood number of nanofluid for varying parameter of chemical reaction when $Sc = 2.5$ and $\varphi = 0.02$.

Parameters	Water	Ethylene glycol
k_o	*Sh*	*Sh*
5	4.90749	4.90923
7.5	7.36124	7.36385
10	9.81498	9.81846
12.5	12.2687	12.2731

Table 6 Sherwood number of ethylene glycol for varying parameters of chemical reaction when $Sc = 2.5$ and $\varphi = 0.02$.

Parameters	Water	Ethylene glycol
Φ	*Sh*	*Sh*
0.02	4.90749	4.90923
0.06	4.72336	4.72876
0.10	4.54052	4.54984
0.14	4.35914	4.37269

Table 7. Nusselt number of the nanofluid samples for varying Prandtl number.

Water	Results of data when Brownian motion is taken into account as $N = 3$ and $\varphi = 0.02$	Results of data when Brownian motion is not taken into account as $N = 3$ and $\varphi = 0.02$
Pr	*Nu*	*Nu*
4	3.46817	3.46817
8	3.47946	3.47946
12	3.47729	3.47729
16	3.4739	3.4739
Ethylene glycol	**Results of data when Brownian motion is taken into account, as $N = 3$ and $\varphi = 0.02$**	**Results of data when Brownian motion is not taken into account as when $N = 3$ and $\varphi = 0.02$**
Pr	*Nu*	*Nu*
4	3.46947	3.46947
8	3.48051	3.48051
12	3.47827	3.47827
16	3.47486	3.47486
Water	**Results of data when Brownian motion is taken into account as $Pr = 4$ and $\varphi = 0.02$**	**Results of data when Brownian motion is not taken into account as $Pr = 4$ and $\varphi = 0.02$**
N	*Nu*	*Nu*
3	3.46817	3.46817
6	6.4332	6.4332
9	9.37464	9.37464
12	12.3144	12.3144
Ethylene glycol	**Results of data when Brownian motion is taken into account, $Pr = 4$ and $\varphi = 0.02$**	**Results of data when Brownian motion is not taken into account**
N	*Nu*	*Nu*
3	3.46947	3.46947
6	6.43523	6.43523
9	9.37764	9.37764
12	12.3184	12.3184
Water	**Results of data when Brownian motion is taken into account as $Pr = 4$ and $N = 3$**	**Results of data when Brownian motion is not taken into account as $Pr = 4$ and $\varphi = 0.02$**
Φ	*Nu*	*Nu*
0.02	3.46817	3.46817
0.06	3.35739	3.35739
0.10	3.24738	3.24738
0.14	3.13825	3.13825
Ethylene glycol	**Results of data when Brownian motion is taken into account as $Pr = 4$ and $\varphi = 0.02$**	**Results of data when Brownian motion is not taken into account as $Pr = 4$ and $\varphi = 0.02$**
Φ	*Nu*	*Nu*
0.02	3.46947	3.46947
0.06	3.36141	3.36141
0.10	3.25429	3.25429
0.14	3.14824	3.14825

that the Nusselt number increases from the second decimal place as the Prandtl number increases. The result also shows that the same numerical value of the Nusselt number is displayed for all values of the Prandtl number when the action of Brownian motion is not taken into account. The same pattern of results was observed from the results presented for both base fluid samples of the nanofluid. The difference between the base fluids is

Table 8. Skin friction of the nanofluid samples for varying thermal Grashof number *N = 3* and φ = *0.02, Sc = 2.5*, k_o = *5.0*, *Re = 100*, *Pr* = 4.*0*, *Grc = 5.0*, *Ha = 10.*

Water	**Results of data when Brownian motion is taken into account,**	**Results of data when Brownian motion is not taken into account**
Gr_θ	*cf*	*cf*
6	-136.2024366716886	-136.20245533106643
12	-272.23372543087436	-272.2337627508987
18	-408.265001881043	-408.2650578583273
24	-544.2962750746457	-544.2963497187111

Ethylene glycol	**Results of data when Brownian motion is taken into account,**	**Results of data when Brownian motion is not taken into account**
6	-9.723585842821421	-9.723594154903063
12	-19.274778054588076	-19.274794679511828
18	-28.825944549078784	-28.82596948742441
24	-38.37709530946198	-38.37712856091223

observed as the Nusselt number obtained for ethylene glycol base fluid is relatively higher than the values of the Nusselt number obtained for the water base nanofluid. Numerical data of the effect of Prandtl number on the Brownian motion reveal that, as the Prandtl number increases, values of the Brownian motion action is kept at a constant value all through. The physical implication of the results in Table 7 for the water base and ethylene glycol base nanofluid is that as the momentum heat transfer increases over thermal heat transfer when the Prandtl number increases, the heat transfer by convection decreases over heat transfer by conduction when the Nusselt number decreases. Brownian motion is also observed to contribute towards the positive values of the Nusselt number obtained, although its value is seen to be constant, as heat transfer by momentum increases over heat transfer by thermal conductivity when the Prandtl number increases. Further implication of the result is that, during heat transfer by the transfer of momentum by particles of base fluid, the domination of the transfer of heat energy by Brownian motion when the fluid is in motion remain the same, even when heat transfer by momentum transport of nanoparticles increases. The result partially agrees with the result presented by Ashwinkumar *et al.* (2021) for the first step increase in Prandtl number, while as the Prandtl number increases further, results are seen to vary.

The result of the Nusselt number for varying parameters of radiation terms for the water-based and ethylene glycol-based nanofluid under various conditions associated with the Brownian motion is presented in Table 7. The result shows that the Nusselt number obtained increases as the parameter of the radiation term increases. This same observation was made when Brownian motion was not taken into account for both base fluid samples. The result also shows that the values of the Nusselt number obtained for ethylene glycol base nanofluid is higher than values of Nusselt number obtained for the water base nanofluid. The physical implication of the results in Table 8 for the water base and ethylene glycol base nanofluid is that, for continuous increase in heat transfer by convection over heat transfer by conduction when the Nusselt number increase on the occasion on increases in the parameter of radiation term. This further implies that in the instance where there is an increase in decay of the absorption coefficient term of the Rosseland radiative energy flux, the rate of heat transfer during fluid motion keeps increasing over the rate of heat transfer when the base fluid is stagnant. This was observed when Brownian motion is taken into account, and when Brownian motion is not taken into account, for the water base and ethylene glycol base nanofluids. The implication of the difference in the result of the Nusselt number for varying parameters of radiation term, between the water base and the ethylene glycol base nanofluid, is that heat transfer during fluid motion is higher in the ethylene glycol base nanofluid than it is in the water base nanofluid. The result of the influence of Brownian motion presented shows that Brownian action contributed to the heat transfer action of the water-based and ethylene glycol-based nanofluid. Although numerical result presented shows no difference in the values of Nusselt number between the values presented when Brownian motion is taken into account and when Brownian motion is not taken into account, hence, no numerical increase is motivated by the presence of Brownian motion, but numerically the contribution of Brownian motion to the heat transfer is not absolutely zero. And the value of heat transfer by Brownian motion during fluid transport increases over heat transfer by Brownian motion when base fluid is stagnant, as the increase in decay absorption is observed. The behavior of heat transfer motivated by Brownian motion is observed from both base fluid samples of nanofluid. The result agrees excellently with the result of Rout and Mishra (2018) under steady-state conditions but varies in an unsteady

Table 9. Skin friction of the nanofluid samples for varying modified Grashofs number $N = 3$ and $\varphi = 0.02$, $Sc = 2.5$, $k_o = 5.0$, $Re = 100$, $Pr = 4.0$, $Grc = 6.0$, $Ha = 10$.

Water	**Results of data when Brownian motion is taken into account,**	**Results of data when Brownian motion is not taken into account**
Gr_C	*cf*	*Cf*
5	-136.2024366716886	-136.20245533106643
10	-136.39345332290154	-136.39347198310742
15	-136.58446884789737	-136.58448750764552
20	-136.77548325310988	-136.77550191403844

Ethylene glycol	**Results of data when Brownian motion is taken into account,**	**Results of data when Brownian motion is not taken into account**
Gr_C	*cf*	*Cf*
5	-9.723585842821421	-9.723594154903063
10	-9.916340450346755	-9.916348762676748
15	-10.109091717201562	-10.109100029481773
20	-10.30183706756798	-10.301845380141945

state, alongside the result of Ashwinkumar *et al*. (2021) and Kumar *et al.* (2022).

The result of the Nusselt number presented for varying values of the nanoparticle volume fraction when Brownian motion is taken into account, when Brownian motion is not taken into account is presented in Table 9 for water base and ethylene glycol base nanofluid, respectively. The results in both tables reveal that, as the nanoparticle volume fraction increases, the values of the Nusselt number obtained decrease when Brownian motion is taken into account and when Brownian motion is not taken into account. The result also shows that the Nusselt number obtained for the ethylene glycol base nanofluid is relatively higher than the values of the Nusselt number obtained for water. The physical implications of the results in Table 7 for the water base and ethylene glycol base nanofluid is that, as the number of nanoparticles volume fraction increases, the overall rate of heat transfer decreases as heat transfer by convection decreases over heat transfer by conduction. This further implies that heat transfer when the base fluid is in motion decreases over heat transfer when the base fluid is stagnant and increases as the number of nanoparticles per unit volume increases. This pattern of result is observed from the nanofluid when Brownian motion is taken into account, and when Brownian motion was not taken into account for the water base nanofluid and ethylene glycol base nanofluid. The implication of the result on the difference in values of the Nusselt number between the two base fluids presented in Table 7 is that with the same volume of nanoparticle per unit volume of base fluid, heat transfer, heat transfer by convection is greater in ethylene glycol base nanofluid than heat transfer by convection in the water base nanofluid. Heat transfer motivated by Brownian motion decreases when the base fluid is in motion and increases when the base fluid is stagnant as the number of nanoparticles per unit volume of base fluid increases. This physical behaviour is observed in both fluid samples. But the heat transfer is more in the ethylene glycol base nanofluid than in water base nanofluid. The result in this study disagrees with the result of Acharya (2022) but is in good agreement with the result of Jalili *et al*. (2022) and Patil *et al.* (2018).

The results of the skin friction of the water and ethylene glycol base Tin oxide nanofluid for increasing the value of the thermal Grashof number is presented in Table 8. The values of the skin friction obtained shows that as the thermal Grashofs number increases, values of the nanofluid skin friction decreases, both when Brownian motion is taken into account and when Brownian motion is not taken into account. The same observation is made for both the water-based and ethylene glycol-based nanofluid. However, when the values of the skin friction of both base fluid samples were compared, it was observed that the ethylene glycol base nanofluid is at a higher skin friction than the water base nanofluid. This physical implication of the result obtained is that, flow condition in spherical enclosures in which buoyancy force dominates viscous force, skin friction will keep diminishing as buoyancy force increases further. This result disagrees with the result of Ashwinkumar *et al.* (2021)

The result of the skin friction of the water-based nanofluid for increasing the value of the modified Grashof number is presented in Table 9. The result shows that as the modified Grashof number increases, the value of skin friction decreases when Brownian motion is taken into account and when Brownian motion is not taken into account. The results also show that Brownian motion causes a slight increase in skin friction for the water-based nanofluid. The same pattern of results is observed from the values of skin friction presented in Table 9 for the ethylene glycol base nanofluid, which is also seen to be at a relatively higher value of skin friction.

The results of the skin friction obtained when the

Table 10. Skin friction of the nanofluid samples for varying chemical reaction term. *N = 3* and *φ = 0.02, Sc = 2.5*, *Pr = 5.0*, *Re = 100*, *Gr*$_\theta$ *= 6.0*, *Grc = 5.0*, *Ha = 10.*

Water	Results of data when Brownian motion is taken into account,	Results of data when Brownian motion is not taken into account
k$_o$	*cf*	*Cf*
5	-136.2024366716886	-136.20245533106643
7.5	-136.15817409948278	-136.15819275837345
10	-136.12757044037753	-136.12758913750537
12.5	-136.1067181636141	-136.10673693435123

Ethylene glycol	Results of data when Brownian motion is taken into account,	Results of data when Brownian motion is not taken into account
k$_o$	*cf*	*cf*
5	-9.723585842821421	-9.723594154903063
7.5	-9.678862535710612	-9.6788708482082
10	-9.647972542970729	-9.647980860369312
12.5	-9.626933924628169	-9.62694223770688

Table 11. skin friction of the nanofluid samples for varying Hartmann number *N = 3* and *φ = 0.02, Sc = 2.5*, *Pr = 5.0*, *Re = 100*, *Gr*$_\theta$ *= 6.0*, *Grc = 5.0*, *k*$_o$ = 5.0.

Water	Results of data when Brownian motion is taken into account,	Results of data when Brownian motion is not taken into account
Ha	*cf*	*cf*
10	-136.2024366716886	-136.20245533106643
20	-75.32715920041693	-75.32716998261598
30	-53.419743432876345	-53.419751227307934
40	-41.8714244628837	-41.871430628476496

Ethylene glycol	Results of data when Brownian motion is taken into account,	Results of data when Brownian motion is not taken into account
Ha	*cf*	*cf*
10	-9.723585842821421	-9.723594154903063
20	-5.381955462798339	-5.3819602684142875
30	-3.8166493393180256	-3.816652814418145
40	-2.990472403372018	-2.990475152969195

parameter of chemical reaction is increasing is presented in Table 10 for the water-based and ethylene glycol base nanofluid. The result presented shows decreasing negative values of skin friction, which imply increasing values of skin friction for increasing chemical reaction terms. Values of the skin friction presented in the tables for ethylene glycol-based nanofluid show relatively higher values of skin friction over the water-based nanofluid.

The physical implication of the result is that, as the parameter of chemical reaction increases, the skin friction increases proportionately, thereby resulting in a decrease in flow velocity. This is why the water-based nanofluid is seen at a lower skin friction than the ethylene glycol-based nanofluid. Result of the skin friction presented in Table 10 and 11 also reveal that Brownian motion tends to oppose the establishment/increase of skin friction for each increase in parameter of chemical reaction, because the skin friction when Brownian motion is taken into account is observed to be relatively less than the skin friction when Brownian motion is not taken into account. The same pattern of results is obtained for both samples of nanofluid in this study. The result is in alignment with the result of Rout and Mishra. (2018) under an unsteady state, but contrary in steady state condition.

Table 11 presented values of the skin friction for increasing values of the Hartmann number for the water and ethylene glycol base Tin oxide nanofluid, respectively. The result presented shows that as the magnetic Hartmann number increases, the values of the skin friction obtained also increases in a flow situation

Table 12. Skin friction of the nanofluid samples condition for varying Prandtl number $N = 3$ and $\varphi = 0.02$, $Sc = 2.5$, $k_o = 5.0$, $Re = 100$, $Gr_\theta = 6.0$, $Grc = 5.0$, $Ha = 10$.

Water	Results of data when Brownian motion is taken into account,	Results of data when Brownian motion is not taken into account
Pr	*cf*	*cf*
4	-136.2024366716886	-136.20245533106643
8	-137.57788860255374	-137.57789640102678
12	-137.96583721683143	-137.96584178963235
16	-138.13922700832296	-138.1392301465219

Ethylene glycol	Results of data when Brownian motion is taken into account,	Results of data when Brownian motion is not taken into account
Pr	*cf*	*cf*
4	-9.723585842821421	-9.723594154903063
8	-9.819156980633457	-9.819160449443002
12	-9.846083600981613	-9.846085633885703
16	-9.858112859776798	-9.858114253511065

Table 13. Skin friction of the nanofluid samples for varying Radiation term $Pr = 4$ and $\varphi = 0.02$, $Sc = 2.5$, $k_o = 5.0$, $Re = 100$, $Gr_\theta = 6.0$, $Grc = 5.0$, $Ha = 10$.

Water	Results of data when Brownian motion is taken into account,	Results of data when Brownian motion is not taken into account
N	*cf*	*cf*
3	-136.2024366716886	-136.20245533106643
6	-116.39054669852518	-116.39054547173227
9	-104.71379509614579	-104.71379104284233
12	-97.7224541519744	-97.72244980187591

Ethylene glycol	Results of data when Brownian motion is taken into account,	Results of data when Brownian motion is not taken into account
N	*cf*	*cf*
3	-9.723585842821421	-9.723594154903063
6	-8.332994270927003	-8.3329937246027
9	-7.517288645570837	-7.517286828100508
12	-7.027753347036087	-7.027751399533058

when Brownian motion is taken into account, and when Brownian motion is not taken into account for both samples of nanofluid. The result obtained also shows that Brownian motion causes a decrease in the skin friction when the skin friction of both flow conditions is compared.

The physical implication of the result presented here is that, as the effect of the Lorentz drag increases, the skin friction of the nanofluid samples increases proportionately. The values of the skin friction obtained from the water base and ethylene glycol base nanofluid reveal that the skin friction of the ethylene glycol base nanofluid is relatively higher, which consequently implies that it will be at a lower velocity value than the water base nanofluid. The result agrees excellently with the result of Farooq *et al*. (2024) and Kumar *et al*. (2022), whose works also report an increase in skin friction as the Lorentz force increases. But disagrees with the reports of Sarkar *et al*. (2023) and Acharya (2022).

The result of skin friction obtained for the water base and ethylene glycol base nanofluid is presented in Table 12 for varying dimensionless Prandtl numbers. Results show that as the Prandtl number increases, the value of the skin friction is decreasing by becoming more negative. And the value of the skin friction when Brownian motion is taken into account is observed to be greater than the skin friction when Brownian motion is taken into account.

The result here implies that Brownian motion tends to increase the skin friction even as its value is decreased for increasing Prandtl number. The physical implication of the result obtained is that, as the Prandtl number increases, the viscous characteristic of the nanofluid increases while its thermal properties decrease; the increase in the viscous property explains why there is a decrease in the velocity profile. The result aligns with the result of Ashwinkumar *et al.* (2021).

Table 14. Skin friction of the nanofluid samples for varying Reynolds number $N = 3$ and $\varphi = 0.02$, $Sc = 2.5$, $k_o = 5.0$, $Pr = 4.0$, $Gr_\theta = 6.0$, $Grc = 5.0$, $Ha = 10$.

water	**Results of data when Brownian motion is taken into account,**	**Results of data when Brownian motion is not taken into account**
Re	*cf*	*cf*
100	-136.2024366716886	-136.20245533106643
200	-136.16363803075902	-136.16365692426814
300	-136.15075381303404	-136.15077278409365
400	-136.14432280162475	-136.14434181089172

Ethylene glycol	**Results of data when Brownian motion is taken into account,**	**Results of data when Brownian motion is not taken into account**
Re	*cf*	*cf*
100	-9.723585842821421	-9.723594154903063
200	-9.729715761562009	-9.72972418070895
300	-9.731753168034905	-9.731761622723589
400	-9.732770923217728	-9.732779395591479

Table 15. skin friction of the nanofluid samples for varying Schmidt number $N = 3$ and $\varphi = 0.02$, $Pr = 4.0$, $k_o = 5.0$, $Re = 100$, $Gr_\theta = 6.0$, $Grc = 5.0$, $Ha = 10$.

Water	**Results of data when Brownian motion is taken into account,**	**Results of data when Brownian motion is not taken into account**
Sc	*cf*	*Cf*
2.5	-136.2024366716886	-136.20245533106643
3.5	-136.1991822995645	-136.19920096005927
4.5	-136.19717496312455	-136.19719362452966
5.5	-136.1958213156302	-136.19583997579045

Ethylene glycol	**Results of data when Brownian motion is taken into account,**	**Results of data when Brownian motion is not taken into account**
Sc	*cf*	*Cf*
2.5	-9.723585842821421	-9.723594154903063
3.5	-9.720264082189514	-9.720272394402942
4.5	-9.718215977481494	-9.718224289500771
5.5	-9.716835135008909	-9.716843447134654

The results of the skin friction for the water-based and ethylene glycol-based nanofluid are presented in Table 13. Results presented show that the skin friction increases by becoming less negative as the parameter of radiation term increases. The skin friction of the ethylene glycol base nanofluid displays much higher values of skin friction than the water base nanofluid, which implies that the ethylene glycol base nanofluid experiences more flow resistance from the solid wall of the spherical enclosure than the water base nanofluid.

Comparation of the values of skin friction presented in the tables when Brownian motion is taken into account and when Brownian motion is not taken into account revealed that the presence of Brownian motion decreases the value of skin friction. The report of Kumar *et al.* (2022) shows that skin friction is independent of Brownian motion and the radiation term, which is different from the result that was obtained in this study.

The values of skin friction for varying values of Reynold's number are presented in Table 14. The values of the result show a decrease in the value of skin friction for increasing the value of Reynolds number. The increase in skin friction is observed when Brownian motion is taken into account and when it is not taken into account. The action of Brownian motion is also seen to cause a decrease in the skin friction of nanofluid in a spherical enclosure.

The result of the skin friction for varying values of the Schmidt number presented in Table 15 for the water base and ethylene glycol base nanofluid shows a pattern of results where the skin friction is increasing as the Schmidt number increases. The increase in skin

Table 16. skin friction of the nanofluid samples for varying nanoparticle volume fraction. $N = 3$ and $Sc = 2.5$, $Pr = 4.0$, $k_o = 5.0$, $Re = 100$, $Gr_\theta = 6.0$, $Grc = 5.0$, $Ha = 10$.

Water	Results of data when Brownian motion is taken into account	Results of data when Brownian motion is not taken into account
φ	*Cf*	*Cf*
0.02	-136.2024366716886	-136.20245533106643
0.06	-347.2070312358928	-347.2071046163497
0.10	-506.7199742283544	-506.72011439034105
0.14	-633.3911655662871	-633.3913838713707

Ethylene glycol	Results of data when Brownian motion is taken into account	Results of data when Brownian motion is not taken into account
φ	*Cf*	*Cf*
0.02	-9.723585842821421	-9.723594154903063
0.06	-23.86968407814759	-23.869715706118633
0.10	-34.7892723318048	-34.78415893676062
0.14	-43.59258234337178	-43.592675867256744

friction is also observed when Brownian motion action is not taken into action. The skin friction of the ethylene glycol-based nanofluid is seen to be at relatively higher values of skin friction than the water-based nanofluid.

The result of skin friction for varying values of the Tin oxide nanoparticles is presented in Table 16 for water base and ethylene base nanofluid, respectively. The results reveal how an increase in the nanoparticles' volume fraction causes a corresponding increase in the skin friction when Brownian motion is taken into account, when Brownian motion is not taken into account, and when only Brownian motion is considered for all nanofluid samples. The result shows a slight decrease in the skin friction in the occasion where Brownian motion is taken into account, indicating that Brownian motion is responsible for the decrease in skin friction. The result of Kumar *et al*. (2022) reported in their study indicates that skin friction is independent of the Schmidt number.

The physical implication of the results presented is that, as the number of Tin oxide nanoparticles per unit volume increases, the skin friction between the nanofluid and the wall of the spherical enclosure decreases, which could thereby increase flow velocity. This observation is the same for the results of both water-based and ethylene glycol-based nanofluid. The result between the two base fluids reveals that ethylene glycol base nanofluid is at a relatively lower skin friction because of its less negative value. Brownian is also observed to cause a slight increase in the skin friction of both fluid samples as the nanoparticle volume fraction increases. However, a contrary result was reported by Ojjela *et al.* (2022) and Farooq *et al.* (2024).

Conclusion

The comparative study of heat and mass transfer efficiency of the water base and ethylene glycol base nanofluid in MHD spherical enclosure was carried out using combined models of the effective thermal conductivities and effective viscosity, and the radiative heat flux vector approximated by Rosseland. The following are the key conclusions from this study.

- The skin friction increases as nanoparticle volume fraction, magnetic Hartmann number, parameter of radiation, chemical reaction term, flow Reynolds number, and Schmidt number all increase and decrease as the thermal Grashof number and modified Grashof number increase.
- The Nusselt number increases as nanoparticle volume fraction and the parameter of radiation term increases and decreases as the Prandtl number increases.
- The Sherwood number increases as the chemical reaction term and nanoparticle volume fraction increase and is unaffected by increases in the Schmidt number.
- Numerical values of the Nusselt number, skin friction, and Sherwood number of the ethylene glycol base tin oxide nanofluid are higher than the values obtained for the water base tin oxide nanofluid.
- Brownian motion causes a decrease in values of the skin friction at the wall of the spherical enclosure.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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